TEAM ROUND ANSWER KEY

1. There are two ordered pairs, (a, b), of positive integers such that $a < b$ and $a^2 + b^2 = 2020$. Find either pair.

   Answer: $(26, 42)$ OR $(24, 38)$

2. The points $(1, 3)$ and $(7, 11)$ are vertices at opposite ends of a diagonal of the square. Find the coordinates of the other vertex of the square that has $x > 1$.

   Answer: $(8, 4)$

3. An integer is a palindrome if its digits read the same forward or backward. For example, 87,378 is a palindrome. How many integers from 1 to 10,000 are palindromes?

   Answer: 198

4. A swimming pool is in the shape of a circle with diameter 60 ft. The depth varies linearly along the east-west direction from 3 ft at the shallow end in the east to 15ft at the diving end in the west but does not vary at all along the north-south direction. What is the volume of the pool in ft^3?

   Answer: 8100π

5. A bug is flying on a three-dimensional grid and wants to fly from (0, 0, 0) to (2, 2, 2). It flies a distance of 1 unit at each step, parallel to one of the coordinate axes. How many paths can the bug choose which take only six steps?

   Answer: 90

6. Consider the set $A = \{a_1, a_2, a_3, a_4\}$. If the set of all possible sums of any three different elements from A is the set $B = \{-1, 3, 5, 8\}$, then list the elements of the set A.

   Answer: $A = \{-3, 0, 2, 6\}$ (Order does not matter.)

7. How many ordered pairs (a, b) of positive integers are there such that $a \leq 5 \leq b$ and $a, 5, b$ are the lengths of the three sides of a triangle?

   Answer: 15

8. The equation $6^x - 5 \cdot 3^x = 2^{x+2} - 20$ has two real solutions, each of which can be written in the form $x = \log_a(b)$ where a and b are positive integers. Give both solutions.

Answer: $x = \log_3(4), \log_2(5)$

9. Given a cube, construct an octahedron by connecting the centers of each face of the cube (as the vertices of the octahedron). The resulting octahedron has volume that is what fraction of the volume of the cube?

   Answer: 1/6

10. If $a+b+c=7$ and $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{a+c}=\frac{7}{10}$, find the value of $\frac{c}{a+b}+\frac{a}{b+c}+\frac{b}{a+c}$.

    Answer: 19/10

11. Let $\overleftarrow{n}$ denote the digit reversal of the natural number n, so that, for example, $\overleftarrow{123}=321$. Find

$$\Big(10+11+\cdots+99\Big)-\Big(\overleftarrow{10}+\overleftarrow{11}+\cdots+\overleftarrow{99}\Big).$$

    Answer: 405

12. A surveillance drone starts from the origin, (0,0), and flies east with a constant speed of 1 foot per second. After every minute, the drone turns 30° to its left. After 50 minutes, the drone is a distance of $a\sqrt{b+\sqrt{c}}$ feet from the origin, where a, b, and c are positive integers with c having no perfect square factor. Find $a+b+c$.

    Answer: 65 (The drone is $60\sqrt{2+\sqrt{3}}$ ft away.)